



Fisheries and Oceans  
Canada

Science

Pêches et Océans  
Canada

Sciences

**CSAS**

**Canadian Science Advisory Secretariat**

**Research Document 2003/038**

Not to be cited without  
Permission of the authors \*

**SCCS**

**Secrétariat canadien de consultation scientifique**

**Document de recherche 2003/038**

Ne pas citer sans  
autorisation des auteurs \*

**Semi-parametric estimation of tag loss and reporting rates for tag-recovery  
experiments using exact time-at-liberty data**

Noel Cadigan and John Bratley

Department of Fisheries and Oceans  
Science, Oceans and Environment Branch  
P.O. Box 5667  
St. John's, NF  
A1C 5X1

\* This series documents the scientific basis for the evaluation of fisheries resources in Canada. As such, it addresses the issues of the day in the time frames required and the documents it contains are not intended as definitive statements on the subjects addressed but rather as progress reports on ongoing investigations.

Research documents are produced in the official language in which they are provided to the Secretariat.

This document is available on the Internet at:

<http://www.dfo-mpo.gc.ca/csas/>

\* La présente série documente les bases scientifiques des évaluations des ressources halieutiques du Canada. Elle traite des problèmes courants selon les échéanciers dictés. Les documents qu'elle contient ne doivent pas être considérés comme des énoncés définitifs sur les sujets traités, mais plutôt comme des rapports d'étape sur les études en cours.

Les documents de recherche sont publiés dans la langue officielle utilisée dans le manuscrit envoyé au Secrétariat.

Ce document est disponible sur l'Internet à:

ISSN 1480-4883  
© Her Majesty the Queen in Right of Canada, 2003  
© Sa majesté la Reine, Chef du Canada, 2003

**Canada**



### **Abstract**

We present a semi-parametric likelihood approach to estimate reporting rates and tag loss rates from the tags returned from capture-recapture studies. Such studies are commonly used to estimate critical population parameters. Tag loss rates are estimated using double tagged animals, while reporting rates are estimated using information from high-reward tags. A likelihood function is constructed based on the conditional distribution of the type of tag returned (low or high reward, single or double tag), given that a tag has been returned. This involves many sparse  $5 \times 1$  tag-return contingency tables, and choosing a good functional form for the tag loss rate is difficult with such data. We model tag loss rates using monotone smoothing splines, and use these nonparametric estimates to diagnose the parametric form of the tag loss rate. The nonparametric methods can also be used directly to model tag loss rates.

### **Résumé**

Nous présentons une démarche de vraisemblance paramétrique pour estimer les taux de déclaration et de perte d'étiquettes à partir des étiquettes récupérées lors des études de capture-recapture. Des études de ce genre sont couramment utilisées pour estimer les paramètres critiques d'une population. Les taux de perte d'étiquettes sont estimées à l'aide d'animaux étiquetés en double, tandis que les taux de déclaration le sont à partir de l'information sur les étiquettes à récompense élevée. La fonction de vraisemblance repose sur la distribution conditionnelle du type d'étiquette retournée (étiquette à récompense faible ou élevée, étiquette simple ou double). Cette démarche nécessite de nombreux tableaux clairsemés de contingence  $5 \times 1$  sur les étiquettes retournées, et il est difficile de choisir une bonne forme fonctionnelle pour les taux de perte d'étiquettes avec de telles données. Nous modélisons les taux de perte d'étiquettes à l'aide des fonctions splines monotones de lissage et utilisons ces estimations non paramétriques pour diagnostiquer la forme paramétrique du taux de perte d'étiquettes. Les méthodes non paramétriques peuvent aussi être utilisées directement pour modéliser les taux de perte d'étiquettes.

# 1 Introduction

The exploitation rate by a commercial fishery (i.e. fraction of fish stock removed) can be estimated using data from tagging experiments. Essentially, the fraction of tagged fish caught in a commercial fishery and returned by fishermen provides an estimate of the fishery exploitation rate. This estimate can then be used, in conjunction with estimates of the total landings by the fishery, to estimate stock size. Tag-recovery experiments can also provide information on other critical population parameters such as migration and growth rates. Tag-recovery experiments, or more generally mark-recapture experiments, are commonly used to study wildlife populations (see Schwarz and Seber 1999).

The analyses of tagging experiments are often more complex than we have described so far, and we address two problems in this paper. The first problem involves estimating tag reporting rates. Although fishermen are encouraged to return tags, not all tagged fish caught in the fishery are reported. Reporting rates can be estimated by various methods (see Pollock, Hoenig, and Jones, 1991); however, there are few mark-recapture studies of fish populations where the study design facilitates the direct estimation of reporting rates. The tagging experiments we consider involved releasing batches of fish with tags of different reward values, with the value of the highest reward sufficient to ensure a reporting rate of one, or very near one. By assuming the reporting rate for high reward tags is one, we can estimate the reporting rates for other types of tags. This method was developed by Henny and Burnham (1976) and discussed by Pollock et al. (2001) in a fisheries context.

The second problem we address is estimating the rate that fish lose their tags. A tag must be attached as firmly as possible, but in a manner that does not affect the survival or catchability of the fish (see Section 2). For various reasons some fish lose their tag, and tag loss causes the number of tagged fish available for capture to decrease with time. Failure to account for this decrease can lead to under-estimation of exploitation rates (see Seber, 1982). There is extensive literature about estimating tag loss for fish and shellfish populations. Some recent references are Fabrizio et al. (1999), Lenarz and Shaw (1997), and Hampton (1997). In our study we have information about tag loss rates from exact times-at-liberty for large numbers of double tagged fish. Parametric estimation of tag loss rates with this type of data has been considered by many authors, including Wetherall (1982), Xiao (1996), Barrowman and Myers (1996), and Xiao et al. (1999).

The main purpose of this paper is to present a nonparametric estimator of the tag loss rate. The estimator is particularly useful for diagnosing the parametric form of the tag loss rate function. We also show how to use the nonparametric method to statistically test the lack-of-fit of a parametric model. In our application, which involved Atlantic cod (*Gadus morhua*), we show that an alternate parametric model is more suitable than the standard model used in other cod tagging studies. The data are described in Section 2.1, where the likelihood is also developed. In Section 2.2 we review the various approaches that have been used for modelling and estimating tag loss rates, and we propose a nonparametric approach in Section 2.3. The results from our data analysis example are given in Section

3, followed by a discussion in Section 4.

## 2 Methods

### 2.1 Data and likelihood

The tagging experiments we consider have been conducted on cod in coastal waters off insular Newfoundland, Canada, during 1997-2002. The experiments were conducted in NAFO subdivision 3Ps and Divs. 3KL (see Cadigan and Bratney, 2000; Fig. 1). Most cod for tagging were captured with hand-lines equipped with feathered hooks, but some trap-caught and otter-trawled cod were also tagged. Live cod caught with hand-lines or otter-trawls were initially placed directly into a 200 gal tank equipped with running sea water. Trap-caught fish were held at the surface, dipped from the trap catch, and tagged and released immediately. The length of each cod (nearest cm) was recorded. Only cod > 45 cm in fork length and in excellent condition were tagged and released. Experienced technicians tagged most of the cod, and individuals undergoing training were carefully observed by trained technicians who checked tags prior to release. Fish were tagged with one or two 6.3 cm t-bar anchor tags (Floy Tag Co., Seattle, Washington) inserted at the base of the first dorsal fin. Double tags were spaced approximately 3 cm apart on the same side of the fish; one tag was inserted in the same location as single tagged fish, which is referred to as position 1. The location of the second tag is referred to as position 2.

A new reward scheme was introduced to encourage those participating in the fishery to return cod tags and recapture information. The reward for returning a standard tag, and any tags from previous studies, was \$10 Can. Cod were tagged with one standard yellow (\$10 reward), two standard yellow (\$20 reward for returning both tags) or one high-reward pink tag (\$100 reward). During initial experiments, tags were applied in the sequence one pink tag, nine single yellow tags, one pink tag, nine double yellow tags. The proportion of high-reward and double tags was reduced starting in 1999. Tags had the value of the reward printed on them, as well as a serial number and return address. The reward scheme was advertised widely by means of posters illustrating the size, color, and position of attachment of tags and reward values; these were sent to all fish plants processing cod in Newfoundland. Plant managers were contacted to confirm that posters were received and prominently displayed and that plant workers processing cod were familiar with the details of the tagging program. Local fisheries officers distributed posters widely among fishing communities. Pre-addressed cod tag return envelopes were sent to processing plants, local fisheries officers, observers, port samplers, and commercial ground-fish license holders. The reward scheme was also advertised on local radio prior to and during the commercial and recreational fisheries. All individuals who returned tags were sent a standard letter describing the date, size, and location where the fish was tagged along with a request to confirm recapture information as well as provide any further recapture details.

We refer to the release of a batch of tagged fish over a 1-2 week period at a specific

site as an experiment ( $x$ ), and the release time as  $t_x$ . We analyze tag-returns from 111 experiments conducted during 1997-2001 and the first three months in 2002. These were experiments which, in addition to single low reward tags, at least some double or high reward tags were used. If only single low or high reward tags were used then subsequent recaptures do not provide information about tag loss or reporting rates (see equation 1). We only used recaptures prior to March 31, 2002 to allow sufficient time for tags to be mailed. The number of fish tagged in these experiments ranged from 2 to 2282, with an average of 444. Twenty-one technicians performed the tagging; however, most of the tagging (68%) was performed by two technicians. For analysis we used one cm length classes. Length is important because the fishery is size-selective. The tagged fish caught ranged in length from 40 cm to 126 cm, although only 10% of recaptures were smaller than 49 cm, and 10% were larger than 84 cm. Let  $M_{jxl}$  denote the number of length  $l$  fish released with type  $j$  tag in experiment  $x$ . The tag types are: single ( $j = S$ ), double ( $j = DD$ ), and high reward ( $j = H$ ). Two additional tag types can be observed at capture: a single tag from position one of a double release ( $j = D1$ ) and a single tag from position two ( $j = D2$ ). We use 8878 returned tags to estimate tag loss and reporting rates. The annual numbers of fish released and returned are presented in Table 1.

Table 1. Annual number of fish released and recaptured. The release year has an  $r$  superscript. S refers to single tags; DD refers to double tags; HH refers to high-reward tags; D1 refers to a recapture with one tag in position one; D2 refers to a recapture with one tag in position two. Only the recaptures during Jan.-Mar. are shown for 2002.

| Year              | Tag Type |    |     |      |      |       | Year              | Tag Type |    |    |      |      |       |
|-------------------|----------|----|-----|------|------|-------|-------------------|----------|----|----|------|------|-------|
|                   | S        | D1 | D2  | DD   | H    | total |                   | S        | D1 | D2 | DD   | H    | total |
| 1997 <sup>r</sup> | 4331     | -  | -   | 3405 | 907  | 8643  | 1999 <sup>r</sup> | 9823     | -  | -  | 1799 | 1620 | 13242 |
| 1997              | 149      | 8  | 12  | 145  | 47   | 361   | 1999              | 875      | 8  | 18 | 121  | 162  | 1184  |
| 1998              | 204      | 25 | 30  | 158  | 63   | 480   | 2000              | 706      | 19 | 29 | 104  | 124  | 982   |
| 1999              | 233      | 35 | 61  | 227  | 77   | 633   | 2001              | 308      | 2  | 22 | 47   | 42   | 421   |
| 2000              | 131      | 18 | 22  | 95   | 34   | 300   | 2002              | 13       | 1  | 1  | 3    | 5    | 23    |
| 2001              | 45       | 8  | 4   | 30   | 4    | 91    | 2000 <sup>r</sup> | 6826     | -  | -  | 1961 | 319  | 9106  |
| 2002              | 6        | 0  | 1   | 2    | 1    | 10    | 2000              | 503      | 11 | 21 | 151  | 18   | 704   |
| 1998 <sup>r</sup> | 3972     | -  | -   | 3993 | 1940 | 9905  | 2001              | 434      | 13 | 29 | 104  | 8    | 588   |
| 1998              | 179      | 13 | 30  | 188  | 144  | 554   | 2002              | 38       | 2  | 2  | 5    | 0    | 47    |
| 1999              | 324      | 46 | 107 | 338  | 261  | 1076  | 2001 <sup>r</sup> | 6681     | -  | -  | 1    | 1311 | 7993  |
| 2000              | 192      | 20 | 53  | 192  | 95   | 552   | 2001              | 554      | 0  | 0  | 0    | 94   | 648   |
| 2001              | 60       | 7  | 15  | 57   | 41   | 180   | 2002              | 24       | 0  | 0  | 0    | 2    | 26    |
| 2002              | 5        | 1  | 1   | 7    | 2    | 16    | 2002 <sup>r</sup> | 319      | -  | -  | 0    | 63   | 382   |
|                   |          |    |     |      |      |       | 2002              | 2        | 0  | 0  | 0    | 0    | 2     |

We model the number of single tagged fish available to the fishery at time  $t$  in region  $h$  as  $M_{jxth}(t) = \Phi_1(t - t_x)\xi_{xtht}M_{jxt}$ ,  $j = S$  and  $H$ , where  $\Phi_1(t - t_x)$  is the fraction of the tagged population that have not lost their tag during  $t - t_x$ , and  $\xi_{xtht}$  is the fraction of the releases that have survived and are present in fishing region  $h$ . If tags on

double tagged fish are lost independently of each other then the expected number of type  $DD$  tagged fish available to the fishery is  $M_{DDxlh}(t) = \Phi_1(t - t_x)\Phi_2(t - t_x)\xi_{xlht}M_{DDxl}$  where  $\Phi_2(t - t_x)$  is the retention probability for the tag at position 2. The expected numbers of  $D1$  and  $D2$  fish are  $M_{D1xlh}(t) = \Phi_1(t - t_x)\{1 - \Phi_2(t - t_x)\}\xi_{xlht}M_{DDxl}$  and  $M_{D2xlh}(t) = \Phi_2(t - t_x)\{1 - \Phi_1(t - t_x)\}\xi_{xlht}M_{DDxl}$ .

It is common to assume that  $\Phi_1 = \Phi_2$  for all  $t$  (e.g. Wetherall, 1982; Barrowman and Myers, 1996; Cadigan and Brattey, 1999); however, we show in Section 3 that this is not reasonable for our cod tagging experiments. Note that Xiao (1996) and Xiao et al. (1999) did not assume that  $\Phi_1 = \Phi_2$ . In Section 3 we also test the assumption that tags are lost independently.

Let  $R_{jxlht}$  be the number of type  $j$  tag-returns at time  $t$  in region  $h$ , and let  $\lambda_{jht}$  be the reporting rate for the  $j$ th tag type. Reporting rates tend to be region and time specific for a variety of reasons; for example, there may be temporal and regional differences in fishermen's awareness of the tagging program and their willingness to return the low reward tags. Note that we do not estimate a different  $\lambda$  for every time interval and region. A much smaller number are estimated for region and time (i.e. year) categories. The expected number of tag-returns is  $E(R_{jxlht}) = \lambda_{jht}\mu_{lht}M_{jxlh}(t)$  where  $\mu_{lht}$  is the fishery exploitation rate. For high reward tags we assume that  $\lambda_H = 1$  which enables us to estimate the reporting rates for low reward tags. We have no confirmed reports, and very few unconfirmed reports, of fishermen not returning a high reward tag so this assumption seems reasonable. We have also assumed that the exploitation of tagged fish does not depend on the type of tag, which is reasonable for populations like Atlantic cod.

For estimation and inference we use the conditional distribution of  $R_{jxlht}$  given  $R_{xlht} = \sum_j R_{jxlht}$ , which we assume is Multinomial and involves only  $\Phi$  and  $\lambda$ 's. In applications like ours, the marginal distribution of  $R_{xlht}$  contains little information about  $\Phi$  or  $\lambda$ 's and the conditional distribution is appropriate to use; however, this is not always the case. The kernel of the Multinomial loglikelihood function is  $L(\lambda, \Phi) = \sum_{jxlht} r_{jxlht} \log(p_{jxlht})$  where  $p_{jxlht} = \Pr(R_{jxlht} | R_{xlht})$ . This is also the likelihood suggested by Barrowman and Myers (1996). We assume that the reporting rate for double tags (denoted as  $\lambda_d$ ) is greater than the reporting rate for single low-reward tags (denoted as  $\lambda_s$ ) because of the higher reward (\$20) for returning two tags. The tag type probabilities are

$$p_{jxlht} \propto \begin{cases} \lambda_{shl}\Phi_1(t - t_x)M_{Sxl}, & j = S, \\ \lambda_{shl}\Phi_1(t - t_x)\{1 - \Phi_2(t - t_x)\}M_{DDxl}, & j = D1, \\ \lambda_{shl}\Phi_2(t - t_x)\{1 - \Phi_1(t - t_x)\}M_{DDxl}, & j = D2, \\ \lambda_{dhl}\Phi_2(t - t_x)\Phi_1(t - t_x)M_{DDxl}, & j = DD, \\ \Phi_1(t - t_x)M_{Hxl} & j = H. \end{cases} \quad (1)$$

The proportionality  $\propto$  is such that  $\sum_j p_{jxlht} = 1$ .

We pool tag-returns over weekly intervals to simplify our analyses, but we still treat the data as exact time-at-liberty measurements. We feel this is reasonable because within week variations in tag loss, exploitation, and migration rates are usually small. Pooling over longer time scales is less reasonable because the cod fisheries around Newfoundland

can have abrupt changes in effort due to fishery closures, bad weather, etc. Also, migration rates may vary seasonally. If we pooled over longer time periods then the tag-type Multinomial distribution may involve other complicated parameters such as  $\mu_{lht}$  and  $\xi_{xlht}$ . Problems with pooled data have been considered in more detail by Xiao (1996).

## 2.2 Review of tag loss models

Any tag loss model  $\Phi(t)$  must be nonnegative for all  $t$  and must follow the monotonicity constraint  $\Phi(t_1) - \Phi(t_2) \leq 0$  for all  $t_1 \geq t_2$ . One approach is to estimate the  $\phi_i$ 's in  $\Phi(t) = \prod_{i=1}^t \phi_i$ . Provided that  $0 \leq \phi_i \leq 1$  for all  $i$  then  $\Phi(t)$  modelled this way is monotonic decreasing. This approach was used by Sprankle, Boreman, and Hestbeck (1996) and Fabrizio et al. (1999). It is flexible but computationally cumbersome for our data because tagged fish have been recaptured up to 249 weeks since release. This means we would have to estimate 249  $\phi_i$ 's. We also feel that  $\Phi(t)$  varies smoothly with  $t$  and estimating a  $\phi$  each week is not necessary.

Parametric models are more commonly used for  $\Phi$ , such as  $\Phi(t) = \exp(-\phi t)$ . This is a proportional model because  $\Phi(t+1)/\Phi(t) = \exp(-\phi)$  for all  $t$ . Define the mean tag loss rate as  $\Gamma(t) = \Phi^{-1}(t) \frac{d}{dt} \{1 - \Phi(t)\} = -\frac{d}{dt} \log\{\Phi(t)\}$ . For the proportional model  $\Gamma(t) = \phi$  is constant. This is the only model we are aware of that has been used to model tag loss for cod (see Barrowman and Myers, 1996; Myers et al., 1996; Myers, Barrowman, and Hutchings, 1997; Otterå, Kristiansen, and Svåsand, 1998; Julliard et al., 2001).

Models with non-constant tag loss rates are also used, for two reasons. The first reason is that for some species the tag loss rate seems to decline with time because scar tissue develops around the tagging wound, and this holds the tag firmly in place (e.g. Kirkwood, 1981). This phenomena has led to the development of tag loss functions whose mean tag loss rate can decline to zero. For example, Kirkwood (1981) proposed the model

$$\Phi(t) = \beta_2 \{\beta_0 / (\beta_0 + \beta_1 t)\}^{\beta_0}; \beta_0, \beta_1, \beta_2 \geq 0, \quad (2)$$

where  $\beta_2$  is a parameter for immediate tag loss. For this model  $\Gamma(t) = \beta_0 \beta_1 / (\beta_0 + \beta_1 t)$ , and  $\Gamma(t) \rightarrow 0$  as  $t \rightarrow \infty$ . The second reason is that tag loss rates seem to increase with time for some species. This may occur because of degradation of the tagging material or because of tag fouling (e.g. Ebener and Copes, 1982). Also, for some species and types of tags, the tagging wound may get worse over time, and lead to increased loss rates (e.g. Stobo and Horne, 1994). Weatherall (1982) developed a model in which  $\Gamma(t)$  could increase or decrease, and linear models for  $\Gamma(t)$  have been used (e.g. Xiao et al., 1999).

## 2.3 Nonparametric estimator of tag loss rates

The tagging wound in the cod from our experiments appears to heal after approximately 15 weeks and scar tissue develops around the wound, firmly anchoring the tag in place. It seems reasonable that the tag loss rate would decline after scar tissue is formed. However, after a further period of time the tag loss rate may increase again because of degradation

of the tagging material, although it is uncertain how long this may take. To allow for these possibilities we need a flexible monotone model.

Diagnosing the parametric form of the tag loss function using graphical methods is difficult. This is because the likelihood function,  $L(\lambda, \Phi)$ , is based on  $5 \times 1$  contingency tables for the type of tag-recapture ( $j = S, D1, D2, DD, H$ ) each week ( $t$ ) and capture region ( $h$ ), and for each experiment ( $x$ ) and length class ( $l$ ) of fish that were released. For our data there are no "replicates", so each table represents the outcome for a single recaptured fish; hence, there are 8878 sparse contingency tables from which tag loss rates can be estimated. Plotting the zeros and ones versus time-at-liberty is not informative about the shape of the tag loss function. A similar problem exists when using regression models for binary data, and Copas (1983) suggested using nonparametric kernel regression to assist in diagnosing the form of the regression function. We use a similar approach here; however, we do not use a kernel estimator because of the monotonicity constraints on  $\Phi_1(t)$  and  $\Phi_2(t)$ , and because our model also contains reporting rate parameters.

Regression splines (see Ch.6 in Eubank, 1999) offer a relatively easy method for non-parametric regression in a semi-parametric model, although for monotonic functions direct estimation using spline methods can involve complex constrained optimizations. Ramsay (1998) presented an approach that uses only simple unconstrained optimization techniques. We adapt Ramsay's (1998) approach to estimate  $\Phi(t)$  for position 1 and 2 with multinomial data. We defer the development of the spline model to the Appendix; however, the models we use for  $\Phi(t)$  are always positive, at least first order differentiable, and  $\Gamma(t)$  may increase and/or decrease.

### 3 Results

First we estimate reporting rates and tag loss rates using the tag loss model  $\Phi(t) = \exp(-\phi t)$ . This gives a benchmark to assess the monotone spline estimator of  $\Phi$ , and allows us to illustrate why we think the proportional model is not good for cod tagging studies. Reporting rates are estimated for three geographic regions: 3KL - NAFO Divisions 3K and 3L; 3Ps.NO: NAFO Subdivision 3Ps, and Divisions 3O and 3N; 3Pn.4RS: NAFO Subdivision 3Pn, and Divisions 4R and 4S (see Figure 1 in Cadigan and Brattey, 2000). Preliminary analyses also suggested that reporting rates have increased over time. For example, we estimated single tag reporting rates from only the  $S$  and  $H$  returns using logistic regression. A likelihood ratio test that the year effects in reporting rates were zero had a small p-value (0.015). However, regional differences in year effects were not significant (p-value = 0.439), so for simplicity we estimate years effects that are common to all regions; that is, region and year effects are additive.

Maximum likelihood estimates (mle's) of the region and year effects in reporting rates are presented in Table 2. The reporting rate estimate for a region and year combination is the sum of the corresponding region and year effects. We pooled the year effects for 1997 and 1998 at the reference value of zero because of the relatively small number of

tag returns in 1997 (361, see Table 1). Standard errors are computed using the inverse of the Hessian matrix. The constraint  $\lambda_s \leq \lambda_d$  was met for the 3KL region, which is why we do not report a standard error for  $\hat{\lambda}_s$ . Note that reporting rates cannot be greater than one, and this constraint was met for the 2001 and 2002  $\lambda_d$  year effects. We further consider the boundary constraints below. The estimates  $\hat{\phi}_1 = 0.0034$  and  $\hat{\phi}_2 = 0.0020$  are significantly different (likelihood ratio p-value  $< 0.0001$ ).  $\hat{\Phi}_1(t)$  and  $\hat{\Phi}_2(t)$  are plotted in Figure 1. The log-likelihood for the fitted model was  $-8530.51$ .

Table 2. Estimated reporting rates (Est) and standard errors (Se)

| Region/<br>Year | Proportional Model |       |             |       | Kirkwood's Model |       |             |       |
|-----------------|--------------------|-------|-------------|-------|------------------|-------|-------------|-------|
|                 | Single Tags        |       | Double Tags |       | Single Tags      |       | Double Tags |       |
|                 | Est                | Se    | Est         | Se    | Est              | Se    | Est         | Se    |
| 3KL             | 0.750              | -     | 0.750       | 0.076 | 0.691            | 0.076 | 0.778       | 0.089 |
| 3Pn_4RS         | 0.397              | 0.128 | 0.712       | 0.242 | 0.327            | 0.121 | 0.713       | 0.237 |
| 3Ps_NO          | 0.710              | 0.054 | 0.725       | 0.059 | 0.654            | 0.052 | 0.740       | 0.062 |
| 1999            | 0.022              | 0.067 | 0.110       | 0.078 | 0.081            | 0.065 | 0.083       | 0.081 |
| 2000            | 0.104              | 0.078 | 0.230       | 0.095 | 0.178            | 0.077 | 0.200       | 0.097 |
| 2001            | 0.173              | 0.084 | 0.250       | -     | 0.292            | 0.085 | 0.222       | -     |
| 2002            | 0.217              | 0.221 | 0.250       | -     | 0.308            | -     | 0.222       | -     |

Chi-square residuals for type  $D1$  and  $D2$  tag returns from double tagged fish are presented in Figure 2. If  $Y_j$  is a binary indicator variable that is one if a  $Dj$  tag ( $j = 1, 2$ ) is returned and zero if two tags are returned, then the chi-square residual is  $(Y_j - p_t) / \{p_t(1 - p_t)\}^{1/2}$  where  $p_t = \{1 - \Phi_j(t)\} / \{1 - \Phi_j(t) + \Phi_j(t)\lambda_d/\lambda_s\}$ . This residual depends on the reporting rates only through the ratio  $\lambda_d/\lambda_s$ . Usually  $\lambda_s \doteq \lambda_d$  so  $p_t \doteq 1 - \Phi_j(t)$ . Fishermen do not know the time-at-liberty; hence, trends in these residuals most likely indicate mis-specification of  $\Phi(t)$  and not mis-specification of  $\lambda_d/\lambda_s$ . The results in Figure 2 suggest that the proportional tag loss model significantly under-predicts tag loss in the first year-at-liberty, and significantly over-predicts tag loss after two years-at-liberty. Averages of residuals are systematically different from zero during these periods.

We conclude from the preceding analysis that tag retention rates may not decrease proportionately; however, the analysis was not that useful for determining a better parametric form for the tag retention function. To assist with this we estimated  $\Phi_1$  and  $\Phi_2$  using the monotone spline method which are described in the Appendix. We used 30 knots for the spline regression, with the knot locations shown in Figure 1. A penalty function was added to  $-2L(\lambda, \Phi)$  to avoid over-fitting. The penalty weight ( $\alpha$ ) was set at 25. This value minimized the GCV statistic, although the GCV curve was very flat (see Appendix). The resulting estimates are also shown in Figure 1, and suggest higher short-term tag loss rates and lower long-term tag loss rates than the proportional model. The change in log-likelihood ( $\Delta_{obs} = 113.16$ ) between the two models is substantial, while the decrease in error degrees of freedom (1.215) is small; that is, based on the usual

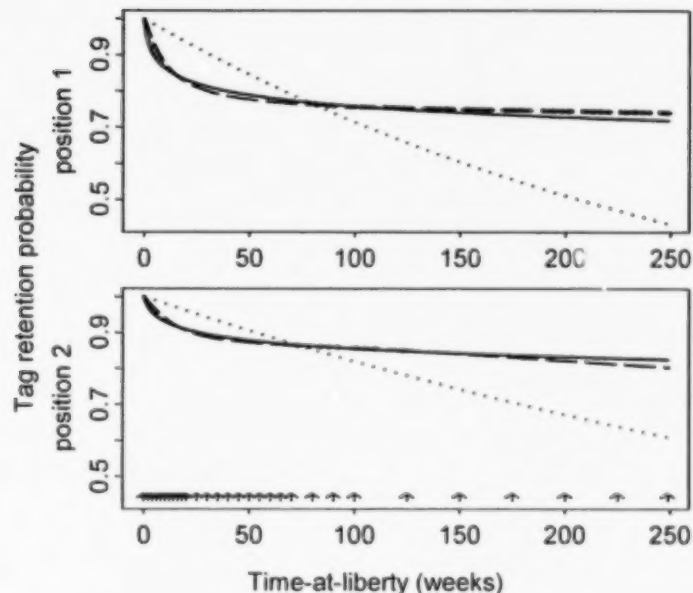


Figure 1: Estimates of  $\Phi_1(t)$  (top panel) and  $\Phi_2(t)$  (bottom panel). The dotted lines are for the model  $\Phi(t) = \exp(\phi t)$ . The solid lines are for Kirkwood's parametric model, and the dashed lines are for the nonparametric estimates. The arrows indicate the location of the spline knots.

chi-square approximation,  $\Pr(\chi_{1,215}^2 > 2\Delta_{obs}) < 0.0001$ . This is a lack-of-fit test for the proportional model versus a nonparametric alternative. The very small p-value means we should reject that the proportional model is suitable. The accuracy of the  $\chi^2$  approximation is questionable; however,  $\Delta_{obs}$  is very large and it seems unlikely that a more accurate approximation to the distribution of  $\Delta$  would change our conclusions. The reporting rate estimates are very similar to those in Table 1 (i.e. Kirkwood's), and are not presented.

In Figure 1 we also show estimates of  $\Phi(t)$  based on (2). The estimates are similar to the nonparametric ones, and the log-likelihood ( $-8418.99$ ) is almost the same; that is,  $\Delta_{obs} = 1.71$ . The lack-of-fit p-value is  $\Pr(\chi_{0.215}^2 > 2\Delta_{obs}) < 0.0092$ , which indicates some model misspecification. However, at this level the accuracy of the  $\chi^2$  approximation must be questioned, although this is beyond the scope of this paper. The chi-square residuals (see Figure 3) are much smaller than those from the proportional model, and do not indicate serious model misspecification. The  $\hat{\lambda}$ 's are shown in Table 2, and are similar to the proportional model estimates.

The year effects in  $\lambda_s$  and  $\lambda_d$  are marginally significant; that is, based on (2) but with and without year effects,  $\Delta_{obs} = 7.9$  and  $\Pr(\chi_8^2 > 2\Delta_{obs}) < 0.045$ . The trend in

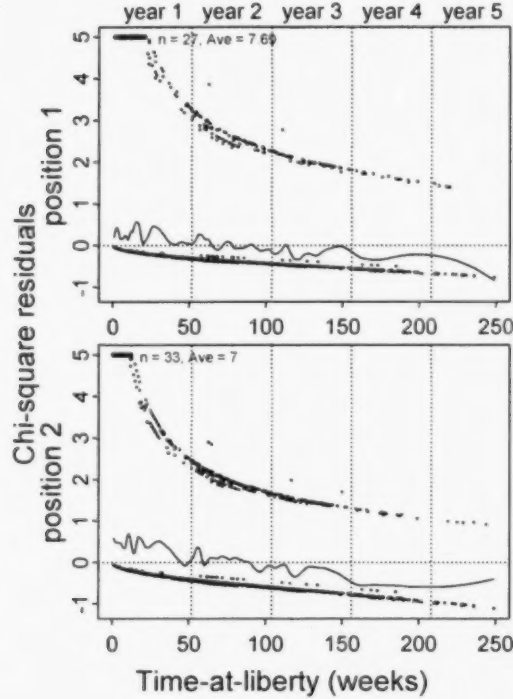


Figure 2: Chi-square residuals for type *D1* (top panel) and type *D2* (bottom panel) tag-returns based on the proportional tag loss model. The residuals are conditional on a captured fish being double tagged at release. The solid line represents a local linear smooth of the residuals. Circles represent truncated residuals. The number and average of the truncated residuals is indicated to the right.

the estimated year effects is realistic and suggests an increasing trend in reporting rates (see Discussion). Differences in  $\Phi_1(t)$  and  $\Phi_2(t)$  are also significant ( $\Delta_{obs} = 36.45$ ; p-value  $< 0.0001$ ). Similar to the proportional model, a number of constraints on the reporting rates have been met in estimation; consequently, some of the standard errors presented in Table 2 may be inaccurate.

Reporting rate estimates at the constraints might indicate model misspecification. Using (2) for  $\Phi(t)$ , we estimated unconstrained reporting rates and found the fit to be almost the same as the constrained fit ( $\Delta_{obs} = 0.6$ ). Differences between constrained and unconstrained estimates were largest in 2002, which is the year for which we have little data (see Table 1). In other years the differences were much smaller, with an average absolute difference of 0.018 for single tags and 0.038 for double tags. We conclude that reporting rate estimates at the constraints simply indicate that they are very close to one in the population.

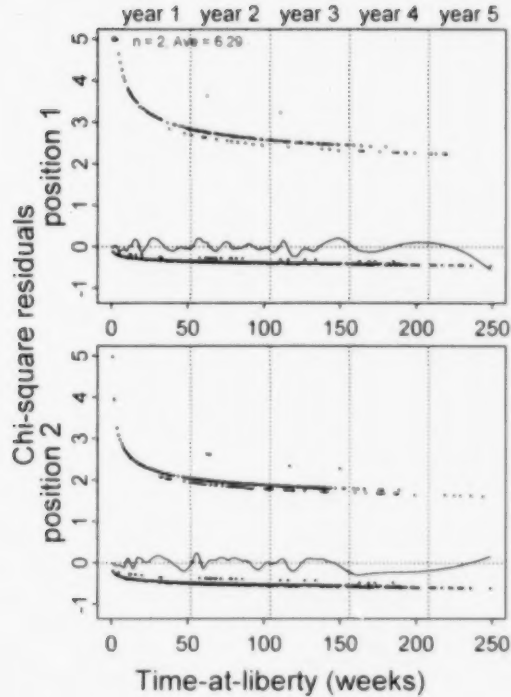


Figure 3: Chi-square residuals for type  $D1$  (top panel) and type  $D2$  (bottom panel) tag-returns based on the Kirkwood's tag loss model. The residuals are conditional on a captured fish being double tagged at release. The solid line represents a local linear smooth of the residuals. Circles represent truncated residuals. The number and average of the truncated residuals is indicated to the right.

Profile likelihood confidence intervals for  $\Phi_1(t)$  and  $\Phi_2(t)$  are shown in Figure 4. The confidence intervals are based on (2). Clearly  $\Phi_1(t)$  and  $\Phi_2(t)$  are significantly different. An advantage of using a parametric model versus the spline method outlined in the Appendix is that more standard procedures exist to construct confidence intervals with parametric models. For the spline approach, Ramsay (1998) mentioned the bootstrap; however, our model involves 76 parameters to estimate, and our data consists of 8878 multinomial outcomes; using the bootstrap in this situation is computationally very cumbersome. Since (2) fits the data almost equally as well as the spline approach, we think the intervals in Figure 4 are sufficient to describe uncertainties about  $\Phi_1(t)$  and  $\Phi_2(t)$  in our application. Developing more feasible confidence interval procedures for the spline method is beyond the scope of this paper, but a useful area for future research.

An important assumption behind the equations for  $M_{DD}$ ,  $M_{D1}$ , and  $M_{D2}$  (see Section 2.1) is that tags are lost independently on double tagged fish. Let  $\Phi_{1|2}(t)$  denote the

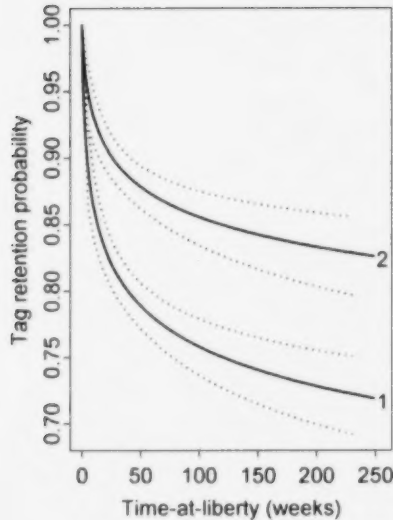


Figure 4: Estimates of  $\Phi_1(t)$  and  $\Phi_2(t)$  using Kirkwood's parametric model (solid lines), with 95% pointwise profile likelihood confidence intervals (dotted lines). The right-hand numbers indicate tag position.

position 1 tag retention probability at time-at-liberty  $t$  given that the tag at position 2 is still attached. Define  $\Phi_{2|1}(t)$  similarly. Dependence implies that  $\Phi_{1|2}(t) \neq \Phi_1(t)$  and  $\Phi_{2|1}(t) \neq \Phi_2(t)$ . One can obtain the dependent tag loss model by replacing  $\Phi_2$  in  $M_{D1}$  and  $M_{DD}$  with  $\Phi_{2|1}$ , and  $\Phi_1$  with  $\Phi_{1|2}$  in  $M_{D2}$ . Note that  $\Phi_{2|1} = \Phi_{1|2}\Phi_2/\Phi_1$  so the dependent tag loss model involves one additional function,  $\Phi_{1|2}(t)$ . In the likelihood function,  $\Phi_1(t)$  and  $\Phi_2(t)$  appear only as the ratio  $\Phi_2(t)/\Phi_1(t)$  and are not uniquely identified without fixing one of the parameters in (2) for either  $\Phi_1(t)$  or  $\Phi_2(t)$ ; hence, the dependence model only has one additional free parameter. A likelihood ratio test that  $\Phi_{1|2}(t) = \Phi_1(t)$  for all  $t$  had a p-value = 0.7, and this does not provide evidence that  $\Phi_{1|2}(t) \neq \Phi_1(t)$ . Estimates of  $\Phi_{1|2}(t)$  were also very similar to estimates of  $\Phi_1(t)$  based on the independence assumption. Our test for dependence may not be very powerful, so we merely conclude that the dependence in tag loss rates, if it exists, is not strong.

Cadigan and Brattey (1999) included a tag loss effect for gear type in previous analyses of part of the tag-recapture data considered here. The rationale for this was that some tags might be torn from fish as they were landed, particularly when gillnets were used. Similar problems for other species have been reported (e.g. Newman and Hoff, 1998; Ebener and Copes, 1982). We examined the total number of observed and predicted tag returns for each type of tag and each gear type used to capture tags. The most significant difference was for the "unknown" gear category, which is when the capture gear is not reported. From a total of 673 returns from this gear category, 33 were type  $D1$  tags and

our model predicted 22.7. This is only marginally significant based on binomial sampling variation.

We performed a similar analysis to check for tagger effects. Only one technician had a significant difference in total observed and model predicted tag returns. The total number of *D1* and *D2* tag returns for this technician was 16 and 21. Our model predicted 20.8 *D1* returns and 40.6 *D2* returns from a total of 731 returns that were tagged by this individual. Combined, our model estimates 61.4 *D1* + *D2* returns while only 37 were observed. The higher predictions are significant when compared to the total estimated binomial variation; however, this technician tagged only 8.7% of the total number of fish tagged, and we do not feel that the differences warrant further modification to our model.

## 4 Discussion

We have presented a likelihood approach to estimate reporting rates and tag loss rates. The likelihood is based on the conditional distribution of the type of tag returned, and this distribution is free from complicated nuisance parameters in our model (e.g. migration and survival rates, exploitation and natural mortality rates, etc.). A problem with this approach is that it involves many sparse  $5 \times 1$  tag-return contingency tables, and diagnosing the "correct" functional form for the tag loss model is difficult with such data.

We have developed methods to estimate tag loss rates nonparametrically to assist with this diagnosis. The nonparametric tag loss estimator can also be used by itself, although we have not provided methods to compute standard errors and confidence intervals for the semi-parametric estimators of tag loss rates and reporting rates, which is beyond the scope of this paper. However, we have provided sufficient results to perform a lack-of-fit test of a parametric tag loss rate model versus a nonparametric alternative, as was illustrated in the Results section. In that section we used the nonparametric estimator to show that Kirkwood's parametric tag loss model was a reasonable choice. The advantage of using a parametric tag loss model is that fairly standard procedures exist for making inferences, e.g. profile likelihood confidence intervals.

Differences in the tag loss rates for the front and back tags on double tagged fish were highly significant. This is a new finding for tagging experiments where cod are double tagged with the same type of tag. Differences in estimates suggest that the back position tags are shed less frequently than those at the front. These differences could be caused by contact with gillnets, which are the most common gear type used during 1997-2002. Many fish caught have abrasions that suggest the fish had previously encountered gillnets. The front tag is at the position where the body of the fish is thickest, and this tag may be dislodged by the gillnet gear contact more so than the back tag. Our results also suggest that the back position may be the better primary tagging site for single tagged fish, because of the lower tag loss rate at this position.

We estimated an increasing trend in the reporting rates. This could be caused by the increasing numbers of tagged fish caught by fishermen. In 1997-1998 fishermen would

have generally caught only 1-2 tagged fish per year, and some fishermen may have decided that it was not financially worthwhile to return one or two low reward tags. However, more recently fishermen have been catching increasing numbers of tagged fish making it financially more worthwhile to return low reward tags.

The proportional model has been used to model tag loss rates in other analyses of cod tag-return data (see Barrowman and Myers, 1996; Myers et al., 1996; Myers et al., 1997; Otterå et al. 1998; Julliard et al., 2001). Our analyses suggest that (2) is a more suitable parametric tag loss model for cod tagged with Floy t-bar anchor tags. For a simple single pulse fishery, incorrectly modelling tag loss will tend to have a multiplicative effect on exploitation rate estimates. For example, if tag loss is estimated to be 20% when it is actually 30% (a 10% difference) then we will under-estimate exploitation rates by 12.5% of the true value. This is because the exploitation rate is basically estimated as the number of tagged fish caught divided by the number of tagged fish available. The effect of incorrectly modelling tag loss will have a greater consequence as the time-at-liberty of the tagged fish progresses.

We assumed that  $\Phi$  does not depend on fish length, although length effects in tag loss rates have been observed in other fish species (e.g. Newman and Hoff, 1998). We have not pursued length effects in this paper; however, residual analyses suggest that tag loss may be slightly greater for large fish (>90 cm) than small fish (<50 cm). Such fish account for only 8% of the tagged population of cod, and small variations in tag loss for these fish are probably not important for estimating total exploitation rates. The assumption about the reporting rate of high-reward tags also needs careful scrutiny, although we feel it is reasonable for the cod fishery we have considered. Examining and testing for these effects often involves adding a parameter to the model to account for the effect, which is relatively straight-forward in the semi-parametric framework we propose here for diagnosing the parametric form (or modelling) of the tag loss rate function.

## 5 Acknowledgments

We wish to thank Dr. Geoff Evans, Fisheries and Oceans, for helpful discussions about this paper. We also thank the sentinel and commercial fishers and staff of the Fisheries Evaluation, Gadoids, and Commercial Sampling Sections of DFO Science for assisting with the tagging experiments and graduate students working under the Chair of Fisheries Conservation, Memorial University for tagging many of the Placentia Bay cod in 1997; C. George, D. Porter, S. Moulton, P. Upward and S. Chipman diligently collated the release and recapture information.

## 6 References

- Barrowman, N. J. and Myers, R. A. (1996). Estimating tag shedding rates for experiments with multiple tag types. *Biometrics* **52**, 1410-1416.

- Cadigan, N. G. and Bratley, J. (1999). Tag loss and reporting rates for 1997 and 1998 cod tagging experiments in 3Psc and 3KL. *DFO Canadian Stock Assessment Secretariat Research Document* **99/65**.
- Cadigan, N. G. and Bratley, J. (2000). Lower bounds on the exploitation of cod (*Gadus morhua*) in NAFO Subdiv. 3Ps and Divs. 3KL in 1997-1999 from tagging experiments. *DFO Canadian Stock Assessment Secretariat Research Document* **2000/073**.
- Copas, J. B. (1983). Plotting  $p$  against  $x$ . *Applied Statistics* **32**, 25-31.
- Ebener, M. P. and Copes, F. A. (1982). Loss of Floy anchor tags from lake whitefish. *North American Journal of Fisheries Management* **2**, 90-93.
- Eubank, R. L. (1999). *Nonparametric regression and spline smoothing*. 2nd ed. New York: Marcell Dekker, Inc.
- Fabrizio, M. C., Nichols, J. D., Hines, J. E., Swanson, B. L., and Schram, S. T. (1999). Modeling data from double-tagging experiments to estimate heterogeneous rates of tag shedding in lake trout (*Salvelinus namaycush*). *Canadian Journal of Fisheries and Aquatic Science* **56**, 1409-1419.
- Green, T. J. and Silverman, B. W. (1994). *Nonparametric regression and generalized linear models*. London: Chapman & Hall.
- Hampton, J. and Kirkwood, G. P. (1990). Tag shedding by southern bluefin tuna *Thunnus maccoyii*. *Fishery Bulletin* **88**, 313-321.
- Hastie, T. J. and Tibshirani, R. J. (1990). *Generalized additive models*. London: Chapman & Hall.
- Henny, C. J. and Burnham, K. P. (1976). A reward band study of mallards to estimate band reporting rates. *Journal of Wildlife Management* **40**, 1-14.
- Julliard, R., Stenseth, N. C., Gjøsæter, J., Lekve, K., Fromentin, J., and Danielssen, D. S. (2001). Natural mortality and fishing mortality in a coastal cod population: A release-recapture experiment. *Ecological applications* **11**, 540-558.
- Kirkwood, G. P. (1981). Generalized models for the estimation of the rates of tag shedding by southern bluefin tuna (*Thunnus maccoyii*). *Journal du Conseil* **39**, 256-260.
- Lenarz, W. H. and Shaw, F. R. (1997). Estimates of tag loss from double-tagged sablefish, *Anoplopoma fimbria*. *Fishery Bulletin* **95**, 293-299.

- Myers, R. A., Barrowman, N. J., Hoenig, J. M., and Qu, Z. (1996). The collapse of cod in Eastern Canada: the evidence from tagging data. *ICES Journal of Marine Science* **53**, 629-640.
- Myers, R. A., Barrowman, N. J., and Hutchings, J. A. (1997). Inshore exploitation of Newfoundland atlantic cod (*Gadus morhua*) since 1948 as estimated from mark-recapture data. *Canadian Journal of Fisheries and Aquatic Science* **54**(Supplement 1), 224-235.
- Newman, S. P. and Hoff, M. H. (1998). Estimates of loss rates of jaw tags on walleyes. *North American Journal of Fisheries Management* **18**, 202-205.
- O'Sullivan, F., Yandell, B. S., and Raynor, W. J. (1986). Automatic smoothing of regression functions in generalized linear models. *Journal of the American Statistical Association* **81**, 96-103.
- Otterå, H., Kristiansen, T. S., and Svåsand, T. (1998). Evaluation of anchor tags used in sea-ranching experiments with atlantic cod (*Gadus morhua* L.). *Fisheries Research* **35**, 237-246.
- Pollock, K. H., Hoenig, J. M., and Jones, C. M. (1991). Estimation of fishing and natural mortality when a tagging study is combined with a creel survey or port sampling. *American Fisheries Society Symposium* **12**, 423-434.
- Pollock, K. H., Hoenig, J. M., Hearn, W. S., and Calingaert, B. (2001). Tag reporting rate estimation: 1. An evaluation of the high-reward tagging method. *North American Journal of Fisheries Management* **21**, 521-532.
- Ramsay, J. O. (1998). Estimating smooth monotone functions. *Journal of the Royal Statistical Society, Series B* **60**, 365-375.
- Ramsay, J. O. (1988). Monotone regression splines in action. *Statistical Science* **4**, 425-461.
- Schwarz, C. J. and G. A. F. Seber. (1999). Estimating Animal Abundance: Review III. *Statistical Science* **14**, 427-456.
- Seber, G. A. F. (1982). *The estimation of animal abundance and related parameters*. 2nd ed. London: Edward Arnold.
- Sprankle, K., Boreman, J., and Hestbeck, J. B. (1996). Loss rates for dorsal loop and internal anchor tags applied to striped bass. *North American Journal of Fisheries Management* **16**, 461-464.
- Stobo, W. T. and Horne, J. K. (1993). Tag loss in grey seals (*Halichoerus grypus*) and potential effects on population estimates. *Canadian Journal of Zoology* **72**, 555-561.

Wetherall, J. A. (1982). Analysis of double tagging experiments. *Fishery Bulletin* **80**, 687-701.

Xiao, Y. (1996). A general model for estimating tag-specific shedding rates and tag interactions from exact or pooled times at liberty for a double tagging experiment. *Canadian Journal of Fisheries and Aquatic Science* **53**, 1852-1861.

Xiao, Y., Brown, L. P., Walker, T. I., and Punt, A. E. (1999). Estimation of instantaneous rates of tag shedding for school shark, *Galeorhinus galeus*, and gummy shark, *Mustelus antarcticus*, by conditional likelihood. *Fishery Bulletin* **97**, 170-184.

## 7 Appendix

We assume that the reader has some familiarity with regression spline methods, which can be obtained in Eubank (1999) or Green and Silverman (1994). If  $\theta(t)$  is the nonparametric model component in a semi-parametric model then a spline approximation of  $\theta(t)$  may be of the form  $\hat{\theta}(t) = \sum_i \beta_i X_i(t)$ , where  $X_i(t)$ 's are fixed basis functions, and the  $\beta_i$  parameters are estimated. A particular type of basis function is shown in (3); however, many other types exist. The  $\beta_i$ 's can be estimated along with other model parameters using maximum likelihood or some other favored estimation method.

The spline approximation we use for  $\Phi(t)$  is  $\ln\{\Phi_\beta(t)\} = \beta_0 \int_0^t \exp\left\{\sum_{i=1}^k \beta_i X_i(u)\right\} du$ . This ensures that  $\Phi(0) = 1$ , which is consistent with data from cage experiments we have conducted in which only 2 of 719 cod lost a tag during a 5-10 day retention period. We also constrain  $\beta_0 < 0$  so that  $\Phi(t)$  is strictly monotone decreasing in  $t$ . We use order-3 I-spline basis functions for the  $X_i(t)$ 's,

$$X_i(t) = \begin{cases} 0, & t < \xi_i \\ \frac{(t - \xi_i)^2 / \{(\xi_{i+2} - \xi_i)(\xi_{i+1} - \xi_i)\}}{1 - (\xi_{i+2} - t)^2 / \{(\xi_{i+2} - \xi_i)(\xi_{i+2} - \xi_{i+1})\}}, & \xi_i \leq t < \xi_{i+1} \\ 1, & \xi_{i+1} \leq t < \xi_{i+2} \\ 1, & t \geq \xi_{i+2} \end{cases} \quad (3)$$

where  $\xi_1 = 0, \xi_2, \dots, \xi_k, \xi_{k+1} = T$  are the knots and  $T$  is the maximum time-at-liberty. Our approach is based on Ramsay (1998), in which the relative curvature of  $\theta(t) = \log\{\Phi(t)\}$  (i.e.  $w(t) = \theta''(t)/\theta'(t)$ ) is approximated using order-2 M-splines (see Ramsay, 1988).

We select a mesh of knots large enough to yield a good approximation to  $\Phi(t)$ , and use a penalty term to control the amount of smoothness in  $\Phi$ . The penalized fit function we use for estimation is  $l_p(\lambda, \beta) = -2L(\lambda, \Phi) + \alpha P(\Phi_\beta)$  where  $\ln\{\Phi_\beta(t)\}$  is given above,  $L(\lambda, \Phi)$  is given in Section 2.1 and  $P(\Phi_\beta)$  is a roughness measure of  $\Phi_\beta$ . The smoothing parameter  $\alpha$  controls the contribution of  $P(\Phi_\beta)$  to  $l_p$ . We use the penalty suggested by Ramsay (1998),  $P\{\Phi_\beta\} = \int_0^T w^2(t) dt$ . This penalty ensures that  $\Phi_\beta(t)$  is strictly decreasing in  $t$ .

The integral in  $P\{\Phi_\beta\}$  has a quadratic form,  $\beta' \mathbf{D} \beta$ . This is because  $w(t) = \sum_{i=1}^k \beta_i M_i(t)$  where  $M_i(t) = \partial I_i(t)/\partial t$  is an order-2 M-spline; hence,  $w^2(t)$  has a quadratic form in  $\beta$ .

The elements of  $\mathbf{D}$  are given by

$$d_{ii} = \int M_i^2(t)dt = \begin{cases} 4/\{3(\xi_{i+2} - \xi_i)\}, & i < k \\ 4/\{3(\xi_{k+1} - \xi_k)\}, & i = k \end{cases}, \text{ and}$$

$$d_{ii+1} = \int M_i(t)M_{i+1}(t)dt = \begin{cases} 2(\xi_{i+2} - \xi_{i+1})/\{3(\xi_{i+3} - \xi_{i+1})(\xi_{i+2} - \xi_i)\}, & i < k-1, \\ 2/\{3(\xi_{k+1} - \xi_{k-1})\}, & i = k-1. \end{cases}$$

Note that for order-2 M-splines  $M_i(t)M_j(t) = 0$  for all  $j > i + 1$ .

To assist in determining an appropriate smoothing parameter  $\alpha$  we use the generalized cross validation statistic (see Hastie and Tibshirani, 1990),  $GCV(\alpha) = nDEV(\alpha) (n - df)^{-2}$  where  $n$  is the number of tag-returns,  $DEV$  is the deviance, and  $df$  is the spline degrees of freedom (see O'Sullivan, Yandell, and Raynor, 1986). Let  $\mathbf{V}_{n \times n} = \text{Diag}\{p_i(1 - p_i)\}$  where  $p_i$  is the estimated probability that the  $i$ th tag-return is the observed type. Also, let  $\mathbf{\Lambda}' = [\dot{p}_1, \dots, \dot{p}_n]$ , where  $\dot{p}$  is the derivative of (1) with respect to  $\lambda$  and  $\beta$ , and evaluated at  $\hat{\lambda}, \hat{\beta}$ . The  $df$  we use is  $df = \text{trace}\left\{(\mathbf{\Lambda}'\mathbf{V}^{-1}\mathbf{\Lambda} + \alpha\mathbf{D})^{-1}\mathbf{\Lambda}'\mathbf{V}^{-1}\mathbf{\Lambda}\right\}$ . Note that if  $\alpha = 0$  then  $df = \dim(\lambda) + \dim(\beta)$ ; otherwise,  $df$  is less than this amount.